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Constant-Cutoff Approach to Strangeness Dependence in Radiative Decays of Hyperons

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We suggest a quantum stabilization method for the $SU(2)$ σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.*, which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number $B = 1$ sector of the model and show that after the collective-coordinate quantization it admits a stable soliton solution which depends on a single dimensional arbitrary constant. We then study the radiative decays of $J^{\pi} = \frac{3}{2}^{+}$ baryons using the constant-cutoff approach to the $SU(3)$ collective treatment of the Skyrme model for hyperons. Thus we investigate the radiative hyperon decays and the variation of the decay widths with strangeness, showing that the present results are in qualitative agreement with the results obtained using the complete Skyrme model.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_{\pi}^{2}}{16} \operatorname{Tr} \partial_{\mu} U \, \partial^{\mu} U^{+} \tag{1.1}$$

where

$$U = \frac{2}{F_{\pi}} (\sigma + i \, \boldsymbol{\tau} \cdot \boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^{+} = 1$) and F_{π} is the pion-decay constant. In (1.2), $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion isotriplet.

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The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1)

$$\mathcal{L}_{SK} = \frac{1}{32e^2} \mathrm{Tr}[U^+\partial_\mu U, U^+\partial_\nu U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It has been shown by several authors (Adkins *et al.*, 1983; see also Witten, 1979, 1983a,b) that, after the collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\tau \cdot \mathbf{r}_0 F(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated therefore a possibility of building a stable single-baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted. They showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r}, t) = A(t)U_0(\mathbf{r})\, A^+(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4} F_\pi^2 \frac{1}{\chi''(0)}\, a + \frac{1}{2} \frac{[\chi''(0)]^3}{(\pi/4)\, F_\pi^2\, b} J(J+1) \tag{1.5}$$

where

$$a = \int_0^\infty \left[\frac{1}{4} s^2 \left(\frac{d\mathcal{F}}{ds}\right)^2 + 8 \sin^2\left(\frac{1}{4}\mathcal{F}\right)\right] ds \tag{1.6}$$

$$b = \int_0^\infty ds\, \frac{64}{3} s^2 \sin^2\left(\frac{1}{4}\mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \frac{1}{4}\mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5) with respect to the arbitrary dimensional scale parameter $\chi''(0)$ is

$$E = \frac{4}{3}\mathrm{F}_\pi\left[\frac{3}{2}\left(\frac{\pi}{4}\right)^2\frac{a^3}{b}J(J+1)\right]^{1/4} \tag{1.9}$$

Despite the nonexistence of the stable classical soliton solution to the nonlinear σ-models it is possible, after collective-coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by MW is not correct since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2}\sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m \text{ even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy}\cos\left[\frac{\sqrt{7}}{2}\ln(cy) + \alpha\right], \qquad m \text{ odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen appropriately. When $F(0) = -n\pi$ then we want to know which of these two solutions are approached by $F(y)$ when $y \to 0$ $(r \to \infty)$. In order to answer to question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2\,dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, (1.13) is reduced to

$$\int_0^\infty 2y[F'(y)]^2 dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ ($r \to 0$) is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\,dy \tag{1.15}$$

From (1.15) we see that $(F'(y) \to \text{const}$ as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a, b, 1992) I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4) as follows:

$$U(\mathbf{r}) = \exp[i\tau\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991; see also Jain *et al.*, 1989) to construct a stable chiral quantum soliton within the original chiral σ-model. We then study the radiative decays of $J^\pi = \frac{3}{2}^+$ baryons using the constant-cutoff approach to the $SU(3)$ collective treatment of the Skyrme model for hyperons (Dalarsson, 1993, 1995a–d, 1996a–c, 1997a–c). Thus we investigate the radiative hyperon decays and the variation of the decay widths with strangeness, showing that the present results are in qualitative agreement with the results obtained using the complete Skyrme model (Haberichter *et al.*, 1997).

The reason why the cutoff approach to the problem of chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary condition $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991), when a cutoff is introduced, the boundary conditions $F(\varepsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. These authors discussed an interesting analogy with the damped pendulum, showing clearly that as long as $\varepsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by (2.2) in Balakrishna *et al.* (1991). From these asymptotic solutions we immediately see that for $\varepsilon \to 0$ the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach have been been discussed in Dalarsson. (1993, 1995a–d, 1996a–c, 1997a–d)

2. CONSTANT-CUTOFF STABILIZATION

Substituting (1.4) into (1.1), we obtain for the static energy of the chiral baryon

$$E_0 = \frac{\pi}{2} F_\pi^2 \int_{\varepsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\varepsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\varepsilon, \infty]$. The cutoff itself is introduced following Balakrishan *et al.* (1991) as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr} \left(r^2 \frac{dF}{dr} \right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\varepsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \varepsilon(t)]$ now depends implicitly on time t through ε(t). Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^+) \, d^3\mathbf{r} \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t)U_0(\mathbf{r}, t)A^+(t), \qquad U^+(\mathbf{r}, t) = A(t)U_0^+(\mathbf{r}, t)A^+(t) \quad (2.4)$$

where

$$U_0(\mathbf{r}, t) = \exp\{i\tau \cdot \mathbf{r}_0 F[r; \varepsilon(t)] \quad (2.5)$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+)\, d^3\mathbf{r} = -E_0 \quad (2.6)$$

is equal to minus the energy E_0 given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and is equal to

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U\, \partial_0 U^+)\, d^3\mathbf{r} = bx^2\, \mathrm{Tr}[\partial_0 A\, \partial_0 A^+] + c[x(t)]^2 \quad (2.7)$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2 F\, y^2\, dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2 \left(\frac{dF}{dy}\right)^2 y^2\, dy \quad (2.8)$$

with $x(t) = [\varepsilon(t)]^{3/2}$ and $y = r/\varepsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$E_0 = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2 \left(\frac{dF}{dy}\right)^2 + 2\sin^2 F \right] dy \quad (2.9)$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2\, \dot{\alpha}_\nu \dot{\alpha}^\nu \quad (2.10)$$

where $\mathrm{Tr}(\partial_0 A\ \partial_0 A^+) = 2\dot{\alpha}_\nu\dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in Bhaduri (1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \alpha^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1) \quad (2.11)$$

where $\langle \mathbf{J}^2 \rangle = J(J+1)$ is the eigenvalue of the square of the soliton angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{-3/8} \Rightarrow \varepsilon^{-1} = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{1/4} \quad (2.12)$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3}\left[\frac{3}{2}\frac{a^3}{b}J(J+1)\right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \to (\pi/4)F_\pi^2 a$ and $b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-3/2}F_\pi$. However in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions, $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.9)–(2.11) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2, and $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as found by Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ-resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507$ GeV^2 and $b = 0.592$ GeV^2, giving exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = \frac{1}{2}$) the cutoffs are equal to

$$\varepsilon = \begin{cases} 0.22 \text{ fm} & \text{for} \quad F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} & \text{for} \quad F_\pi = 150 \text{ MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect since the cutoffs indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1} we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. RADIATIVE DECAYS OF HYPERONS IN THE SU(3) SKYRME MODEL

3.1. Introduction

As argued in Dalarsson (1997b) and Haberichter *et al.* (1997), the data on the electromagnetic decays of baryons, like the reaction $\Delta \to N\gamma$, are rather limited. The ratio of the electric quadrupole (E2) to the magnetic dipole (M1) amplitude obtained by the $\pi^{0(+)}$-photoproduction experiment at MIAMI (Dalarsson (1997b) is E2/M1 $= (-2.5 \pm 0.2)\%$. For $J = \frac{3}{2}$ to $J = \frac{1}{2}$ transitions, which involve strange baryons, the empirical values for the E2/M1 ratios are still not available. The experiments at CEBAF and Fermilab (Haberichter *et al.*, 1997) will provide additional data on those radiative decays and the pattern of flavor symmetry braeking.

Nevertheless, these transitions have been studied within several models and in Schat *et al.* (1995a) an analysis of the hyperon radiative decays was made within the framework of the bound-state approach (Callan and Klebanov, 1985; Callan *et al.*, 1988) to the Skyrme model (Skyrme, 1961, 1962). In that approach hyperons are modeled as kaons bound in the background of the static soliton field. For the particular case of $\Lambda(1405)$ in the bound-state approach to the complete Skyrme model and in the constant-cutoff treatment of the bound-state approach see Park *et al.* (1991) and Park and Weigel (1992). However, in Dalarsson (1997b) hyperons are alternatively described using the $SU(3)$ collective approach to the constant-cutoff model, where the strange degrees of freedom are incorporated as $SU(3)$ collective excitations of the nonstrange soliton to investigate the transitions $B(J = \frac{3}{2}) \rightarrow \gamma B'(J = \frac{1}{2})$.

In the present section we study the radiative decays of $J^{\pi} = \frac{3}{2}^{+}$ baryons using the constant-cutoff approach to the $SU(3)$ collective treatment of the Skyrme model for hyperons (Dalarsson, 1997b). Thus we investigate the variation of the decay widths with strangeness, showing that the present results are in qualitative agreement with the results obtained using the complete Skyrme model (Haberichter *et al.*, 1997).

3.2. The Effective Interaction

The Lagrangian density for the $SU(3)$ collective model of hyperons is given, with the Skyrme stabilizing term omitted, by (Haberichter *et al.*, 1997)

$$
\begin{aligned}
\mathcal{L} = & \frac{F_{\pi}^2}{16} \operatorname{Tr} D_{\mu}U\, D^{\mu}U^{+} - \frac{F_{\pi}^2 m_{\pi}^2 + 2F_{k}^2 m_{k}^2}{48} \operatorname{Tr}[U + U^{+} - 2] \\
& + \frac{F_{\pi}^2 m_{\pi}^2 - F_{K}^2 m_{K}^2}{8\sqrt{3}} \operatorname{Tr}[\lambda_8 (U + U^{+})] \\
& + \frac{F_{\pi}^2 - F_{k}^2}{16} \operatorname{Tr}\{\bar{S}[U(D_{\mu}U)^{+} D^{\mu}U + U^{+} D_{\mu}U (D^{\mu}U)^{+}]\} \\
& + iL_9 (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}) \operatorname{Tr}[Q(U^{+}\partial^{\mu}U\, U^{+}\partial^{\nu}U + U\partial^{\mu}U^{+}\, U\partial_{\nu}U^{+}]
\end{aligned} \tag{3.1}
$$

where F_K is the kaon decay constant, m_{π} and m_K are the pion and kaon masses, respectively, and

$$\bar{S} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.2}$$

is the projector onto the strange degrees of freedom, and

$$D_\mu U = \partial_\mu U + ieA_\mu[Q, U] \tag{3.3}$$

The last term in (3.1) represents the direct derivative coupling of the baryon fields to the photon field A_μ. In fourth-order chiral perturbation, the last term in (3.1) is necessary to reproduce the electromagnetic pion radius correctly in this model, thus determining the value of the parameter $L_9 = (6.9 \pm 0.7) \times 10^{-3}$. In addition to the action obtained using the Lagrangian (3.1), the Wess–Zumino action gauged to contain the photon field

$$\begin{aligned} S = &-\frac{iN_c}{240\mu^2}\int d^5x\, e^{\mu\nu\alpha\beta\gamma}\,\mathrm{Tr}[U^+\partial_\mu U\, U^+\partial_\nu U\, U^+\partial_\alpha U\, U^+\partial_\beta U\, U^+\partial_\gamma U] \\ &-\frac{eN_c}{48\pi^2}\int d^4x\, e^{\mu\nu\alpha\beta}A_\mu\,\mathrm{Tr}[Q(U^+\partial_\nu U\, U^+\partial_\alpha U\, U^+\partial_\beta U \\ &- U\partial_\nu U^+\, U\partial_\alpha U^+ U\partial_\beta U^+)] + \mathcal{O}(e^2A_\mu^2) \end{aligned} \tag{3.4}$$

must be included into the total action, where N_C is the number of colors in the underlying QCD. The Wess–Zumino action defines the topological properties of the model, important for the quantization of the solitons. In the $SU(2)$ case the Wess–Zumino action vanishes identically and was therefore not present in the discussions of Sections 1 and 2.

From the total action obtained using (3.1) and (3.4) we obtain the following result for the electromagnetic current:

$$\begin{aligned} J_\mu = &-\frac{F_\pi^2}{8}\mathrm{Tr}[Q(U^+\partial_\mu U + U\partial_\mu U^+)] \\ &-\frac{F_\pi^2 - F_K^2}{16}\mathrm{Tr}\{Q[\{U\bar{S} + \bar{S}U^+, U^+\partial_\mu U\} + \{\bar{S}U + U^+\bar{S}, U\partial^\mu U^+\}]\} \\ &- iL_9\,\mathrm{Tr}\{Q\,\partial^\nu([U^+\partial_\nu U^+, \partial_\mu U] + [U\partial_\nu U^+\, U\partial_\mu U^+])\} \\ &-\frac{N_c}{48\pi^2}e_{\mu\nu\alpha\beta}\,\mathrm{Tr}[Q(U^+\partial^\nu U\, U^+\partial^\alpha U\, U^+\partial^\beta U - U\partial^\nu U\, U\partial^\alpha U\, U\partial^\beta U^+)] \end{aligned} \tag{3.5}$$

Following Haberichter *et al.* (1997), we now confine ourselves to static rotations in order to establish the slow rotator approach. Due to spin and isospin invariance this implies the following ansatz for the chiral field:

$$U(\mathbf{r}, \nu) = \exp(-i\nu\lambda_4) \exp[i\tau\cdot\mathbf{r}_0 F(\mathbf{r}, \nu)] \exp(i\nu\lambda_4) \tag{3.6}$$

where the chiral angle depends on the flavor orientation via the strangeness-changing angle $\nu \in [0, \pi/2]$. The time dependence of the flavor rotations is then introduced by introducing the time-dependent meson configuration

$$U(\mathbf{r}, t) = A(t) \exp[i\tau\cdot\mathbf{r}_0 F(r, \nu)]\, A^{+}(t) \tag{3.7}$$

into the total action obtained using (3.1) and (3.4). This gives the Lagrangian as a function of the time derivatives of the collective rotations $A(t)$, which are most conveniently parametrized by introducing the angular velocities ω_a via the expression

$$A^{+}(t)\dot{A}(t) = \frac{i}{2}\sum_{a=1}^{8} \lambda_a\omega_a \tag{3.8}$$

The canonical quantization introduces the right generators of flavor $SU(3)$ as $R_a = -\partial L/\partial\omega_a$ and gives the Hamiltonian of the form

$$H = M(\nu) + \frac{1}{2}\left\{\frac{1}{2\Omega_S(\nu)}, C_2[SU(3)]\right\} + \left[\frac{1}{2\Omega_N(\nu)} - \frac{1}{2\Omega_S(\nu)}\right] J(J+1) - \frac{3}{8\Omega_S(\nu)} \tag{3.9}$$

together with the constraint $R_8 = \sqrt{3}/2$, obtained from the Wess–Zumino term and ensuring that the eigenstates of the Hamiltonian (3.9) have a half-integer spin. In (3.9), $J_m = -R_m$ ($m = 1, 2, 3$) is the spin operator and $C_2[SU(3)]$ is the quadratic Casimir operator of $SU(3)$. The explicit expressions for the inertias $\Omega_N(\nu)$ and $\Omega_S(\nu)$ can be found in Dalarsson (1993, 1995a–d, 1996a–c, 1997a–d). The Hamiltonian (3.9) can be diagonalized exactly and the eigenfunctions are identified as the distorted $SU(3)$ D-function, since in the presence of flavor symmetry breaking, the resulting baryon states are no longer pure octet (for $J = 1/2$) and pure decouplet (for $J = 3/2$) states. They contain sizable admixtures of baryon states in higher dimensional $SU(3)$ representations, like, for example, $\overline{\mathbf{10}}$ or **27** (Haberichter *et al.*, 1997).

Using the covariant form of the electromagnetic current (3.5) and the Hamiltonian (3.9) with the corresponding eigenfunctions, it is possible to obtain the quadrupole and monopole pieces of the electric and magnetic form factors, respectively. The former is obtained from the orbital angular momentum $l = 2$ term of the time component of the electromagnetic current $J_0^{\text{e.m.}}$ and the later is obtained from the spatial components $J_j^{\text{e.m.}}$. It is therefore suitable to introduce the associated Fourier transforms as follows:

$$\hat{E}(q) = \int_{r>\varepsilon} d^3\mathbf{r}\, j_2(qr)\left(\frac{z^2}{r^2} - \frac{1}{3}\right)J_0^{\text{e.m.}} \tag{3.10}$$

$$\hat{M}(q) = \frac{1}{2}\int_{r>\varepsilon} d^3\mathbf{r}\, j_1(qr)\varepsilon^{3ij} r_{0i} J_j^{\text{e.m.}} \tag{3.11}$$

Following Dalarsson (1997b), we obtain in the constant-cutoff approach the results

$$\hat{E}(q) = -\frac{8\pi}{15\Omega_N(\nu)} D_{\text{e.m.},3}\int_{\varepsilon}^{\infty} dr\, r^2 j_2(qr) V_0(r, \nu) \tag{3.12}$$

$$\hat{M}(q) = -\frac{4\pi}{3}\int_{\varepsilon}^{\infty} dr\, r^2 j_1(qr)\Bigg[V_1(r, \nu) D_{\text{e.m.},3} - \frac{F_K^2 - F_\pi^2}{2\sqrt{3}r^2}\sin^2 F\, d_{3\alpha\beta} D_{\text{e.m.}}^{\alpha}\, D_8^{\beta}$$
$$- \frac{1}{2}\left\{\frac{1}{2\Omega_S(\nu)}\frac{1}{4\pi^2 r^2}\sin^2 F\frac{\partial F}{\partial r},\, d_{3\alpha\beta} D_{\text{e.m.}}^{\alpha} R^{\beta}\right\}\Bigg] \tag{3.13}$$

where $\Omega_S(\nu)$ is the moment of inertia in the strange flavor direction ($\alpha, \beta = 4, 5, 6, 7$) and $D_{\text{e.m.},i} = D_{3i} + D_{8i}/\sqrt{3}$.

The functions $V_0(r, \nu)$ and $V_1(r, \nu)$ in (3.13) are given by

$$V_0(r) = \frac{1}{4}\sin^2 F[F_\pi^2 + \sin^2\nu(F_K^2 - F_\pi^2)\cos F]$$
$$- 2L_9\left[\sin 2F\left(\frac{d^2F}{dr^2} + \frac{2}{r}\frac{dF}{dr}\right) + 2\cos 2F\left(\frac{dF}{dr}\right)^2 - 6\frac{\sin^2 F}{r^2}\right] \tag{3.14}$$

$$V_1(r) = \frac{1}{4r^2}\sin^2 F[F_\pi^2 + \sin^2 v(F_K^2 - F_\pi^2)\cos F] \tag{3.15}$$
$$+ \frac{2L_9}{r^2}\left[\sin 2F\frac{d^2F}{dr^2} + 2\cos 2F\left(\frac{dF}{dr}\right)^2 - 2\frac{\sin^2 F}{r^2}\right]$$

3.3. Radiative Decay Widths

The radiative decay widths Γ for the decays of the $\frac{3}{2}^+$ baryons to $\frac{1}{2}^+$ baryons are then obtained as matrix elements of $\hat{E}(q)$ and $\hat{M}(q)$, i.e.,

$$\Gamma_{E2}(B \to \gamma B') = \frac{675}{8}\alpha_{\text{e.m.}}\, q\left|\left\langle B'\left(\frac{1}{2}^+\right)\middle|\hat{E}(q)\middle|B\left(\frac{3}{2}^+\right)\right\rangle\right|^2 \tag{3.16}$$

$$\Gamma_{M1}(B \to \gamma B') = 18\alpha_{\text{e.m.}} q\left|\left\langle B'\left(\frac{1}{2}^+\right)\middle|\hat{M}(q)\middle|B\left(\frac{3}{2}^+\right)\right\rangle\right|^2 \tag{3.17}$$

where we follow the standard prescription (Dalarsson, 1997b) and take q to be the momentum of the photon in the rest frame of the $\frac{3}{2}^+$ baryon, and $\alpha_{\mathrm{e.m.}} = 1/137$. The matrix elements in (3.16) and (3.17) are calculated in the space of collective coordinates; a detailed account can be found in general in Park *et al.* (1991) and Park and Weigel (1992), and in particular for the decays of the $\Lambda(1405)$ resonance in Schat *et al.* (1995b) and Dalarsson (1996b). Now we are able to calculate the desired $E2/M1$ ratio as follows:

$$\frac{E2}{M1} = \frac{5}{4} \frac{|\langle B'(\frac{1}{2}^+)|\hat{E}(q)|B(\frac{3}{2}^+)\rangle|^2}{|\langle B'(\frac{1}{2}^+)|\hat{M}(q)|B(\frac{3}{2}^+)\rangle|^2} \tag{3.18}$$

The numerical predictions of the present model compared with the results obtained using CSM in Haberichter *et al.* (1997) are given in Table I for the same decays as those presented in Dalarsson (1997b). In the present paper we only consider the complete Lagrangian with the third term in (3.1) included, i.e., for $L_9 = 6.9 \times 10^{-3}$, and use the empirical values for pion and kaon masses and decay constants.

Table I Numerical Results for $E2/M1$ Decay Ratios[a]

	$E2/M1$ (%)		
		CSM[b]	
	Present results	HRSW, Table 2	HRSW, Table 3
$\Delta \to \gamma N$	−2.42 (−2.32)	−2.22 (−2.11)	−2.35 (−2.24)
$\Sigma^{*0} \to \gamma\Lambda$	−2.03 (−1.94)	−1.89 (−1.83)	−1.94 (−1.89)
$\Sigma^{*-} \to \gamma\Sigma^-$	−2.21 (−2.11)	−1.95 (−1.91)	−2.34 (−2.29)
$\Sigma^{*0} \to \gamma\Sigma^0$	−1.20 (−1.15)	−1.02 (−1.01)	−1.02 (−1.01)
$\Sigma^{*+} \to \gamma\Sigma^+$	−1.29 (−1.24)	−1.20 (−1.18)	−1.19 (−1.18)
$\Xi^{*-} \to \gamma\Xi^-$	−2.39 (−2.30)	−2.10 (−2.06)	−2.48 (−2.43)
$\Xi^{*0} \to \gamma\Xi^0$	−1.51 (−1.45)	−1.28 (−1.26)	−1.25 (−1.23)

[a] The results shown in parentheses refer to the case when the ratio $E2/M1$ is rescaled by the proton magnetic moment. The rescaling of the type $E2/M1 \to E2/M1 \times (\mu_p^{\mathrm{pre}}/\mu_p^{\mathrm{exp}})$ is motivated by the fact that here, as in the case of the CSM (Haberichter *et al.*, 1997), the predicted value of the proton magnetic moment is lower than the empirical value. For the calculation of the proton magnetic moment in the constant-cutoff approach see Dalarsson (1993, 1995a–d, 1996a–c, 1997a–d) and Schat *et al.* (1995b). Furthermore, note that the numerical results of the constant-cutoff calculations here differ from the constant-cutoff results reported in Dalarsson (1997b) because of the different structure of the symmetry-breaking terms in the Lagrangian (3.1) (Haberichter, 1997) as well as the somewhat different values of the parameters used (Dalarsson, 1997b; Haberichter *et al.*, 1997). We see that the present results are of the same order of magnitude as the CSM results reported in Haberichter *et al.* (1997).

[b] HRSW, Haberichter *et al.* (1997).

4. CONCLUSIONS

We have calculated the decay widths for the radiative decays of the $\frac{3}{2}^{+}$ baryons in the constant-cutoff approach to the collective treatment of the $SU(3)$ Skyrme model by separately evaluating the magnetic dipole (M1) and electric quadrupole (E2) transition matrix elements. As in the CSM (Haberichter *et al.*, 1997) the total decay widths are strongly dominated by the M1 contribution, giving E2/M1 ratios of the order of few percent only. As in the CSM (Haberichter *et al.*, 1997), all the ratios are negative.

We have compared the present results with those obtained using the CSM, showing that there is a general qualitative agreement between our results and the CSM results (Haberichter *et al.*, 1997).

On the other hand, the constant-cutoff approach employed in this paper offers a simpler analytical structure of the results and less complicated calculations of the quantities which describe the strong and electromagnetic properties of hyperons (Dalarsson, 1993, 1995a–d, 1996a–c, 1997a–d).

Finally, it should be noted that the empirical values for most of the calculated quantities are unfortunately difficult to obtain. As argued in Haberichter *et al.* (1997), better empirical information about the radiative decay processes is needed in order to determine the quality of predictions of different models. Some experiments to that effect are being prepared at several experimental facilities (see Haberichter *et al.*, 1997, and references therein).

REFERENCES

Adkins, G. S., Nappi, C. R., and Witten, E. (1983). *Nuclear Physics B*, **228**, 552.

Balakrishna, B. S., Sanyuk, V., Schechter, J., and Subbaraman, A. (1991). *Physical Review D*, **45**, 344.

Bhaduri, R. K. (1988). *Models of the Nucleon*, Addison-Wesley, Reading, Massachusetts.

Callan, C. G., and Klebanov, I. (1985). *Nuclear Physics B*, **262**, 365.

Callan, C. G., Hornbostel, K., and Klebanov, I. (1988). *Physics Letters B*, **202**, 269.

Dalarsson, N. (1991a). *Modern Physics Letters A*, **6**, 2345.

Dalarsson, N. (1991b). *Nuclear Physics A*, **532**, 708.

Dalarsson, N. (1992). *Nuclear Physics A*, **536**, 573.

Dalarsson, N. (1993). *Nuclear Physics A*, **554**, 580.

Dalarsson, N. (1995a). *International Journal of Theoretical Physics*, **34**, 81.

Dalarsson, N. (1995b). *International Journal of Theoretical Physics*, **34**, 949.

Dalarsson, N. (1995c). *International Journal of Theoretical Physics*, **34**, 2129.

Dalarsson, N. (1995d). *Helvetica Physica Acta*, **68**, 539.

Dalarsson, N. (1996a). *International Journal of Theoretical Physics*, **35**, 783.

Dalarsson, N. (1996b). *International Journal of Theoretical Physics*, **35**, 819.

Dalarsson, N. (1996c). *International Journal of Theoretical Physics*, **35**, 2729.

Dalarsson, N. (1997a). *International Journal of Theoretical Physics*, **36**, 487.

Dalarsson, N. (1997b). *International Journal of Theoretical Physics*, **36**, 929.

Dalarsson, N. (1997c). *International Journal of Theoretical Physics*, **36**, 1209.
Dalarsson, N. (1997d). *International Journal of Theoretical Physics*, **36**, 1649.
Haberichter, T., Reinhardt, H., Scoccola, N. N., and Weigel, H. (1997). *Nuclear Physics A*, **615**, 291.
Iwasaki, M., and Ohyama, H. (1989). *Physical Review*, **40**, 3125.
Jain, P., Schechter, J., and Sorkin, R. (1989). *Physical Review D*, **39**, 998.
Mignaco, J. A., and Wulck, S. (1989). *Physical Review Letters*, **62**, 1449.
Park, N. W., and Weigel, H. (1992). *Nuclear Physics A*, **541**, 453.
Park, N. W., Schechter, J., and Weigel, H. (1991). *Physical Review D*, **43**, 869.
Schat, C. L., Gobbi, C., and Scoccola, N. N. (1995a). *Physics Letters B*, **356**, 1.
Schat, C. L., Scoccola, N. N., and Gobbi, C. (1995b). *Nuclear Physics A*, **585**, 627.
Skyrme, T. H. R. (1961). *Proceedings of the Royal Society A*, **260**, 127.
Skyrme, T. H. R. (1962). *Nuclear Physics*, **31**, 556.
Weigel, H., Schechter, J., Park, N. W., and Meissner, U. G. (1990). *Physical Review D*, **42**, 3177.
Witten, E. (1979). *Nuclear Physics B*, **160**, 57.
Witten, E. (1983a). *Nuclear Physics B*, **223**, 422.
Witten, E. (1983b). *Nuclear Physics B*, **223**, 433.